PROVE d ESAME del 12/01/2018

1 (A)

$$\boxed{\lim_{n\to+\infty} (n+2)^{2\alpha+n}\,\frac{(n+3)^{\alpha/2}-\sqrt{n^{\alpha}}}{(n+1)^{n}}}$$

$$= \lim_{n\to+\infty} \left(\frac{n+2}{n+1}\right)^{n}\cdot (n+2)^{2\alpha}\cdot n^{\alpha/2}\left[\left(1+\frac{3}{n}\right)^{\alpha/2}-1\right]$$

$$= \lim_{n\to+\infty} \left(1+\frac{1}{n+1}\right)^{n} n^{2\alpha+\alpha/2}\left[\cancel{1}+\frac{\alpha}{2}\cdot\frac{3}{n}+o\left(\frac{1}{n}\right)\cancel{-1}\right]$$

$$= \lim_{n\to+\infty} e\, n^{2\alpha+\alpha/2-1}\cdot\frac{3\alpha}{2} = \ell$$

Se $2\alpha+\alpha/2-1>0 \iff \alpha>\frac{2}{5} \Rightarrow \ell=+\infty$

Se $2\alpha+\alpha/2-1=0 \quad \alpha=2/5 \Rightarrow \ell=\frac{3}{5}e$

Se $2\alpha+\alpha/2-1<0 \quad 0<\alpha<\frac{2}{5} \Rightarrow \ell=0$

(B)

$$\lim_{n\to+\infty} \frac{\sqrt{(n+1)^{3\beta}}-n^{\frac{3}{2}\beta}}{(n+2)^{n+\beta}}\cdot (n+3)^{n} =$$

$$= \lim_{n\to+\infty} \left(\frac{n+3}{n+2}\right)^{n}\cdot\frac{n^{\frac{3}{2}\beta}\left[\left(1+\frac{1}{n}\right)^{\frac{3}{2}\beta}-1\right]}{(n+2)^{\beta}}$$

$$= \lim_{n\to+\infty} \left(1+\frac{1}{n+2}\right)^{n}\cdot n^{\frac{3}{2}\beta-\beta}\left[\cancel{1}+\frac{3\beta}{2}\frac{1}{n}+o\left(\frac{1}{n}\right)-\cancel{1}\right]$$

$$= \lim_{n\to+\infty} e\cdot n^{\frac{3}{2}\beta-\beta-1}\cdot\frac{3}{2}\beta \Rightarrow \left\{ \frac{3}{2}\beta-\beta-1<0 \right.$$

$\beta<2 \quad \ell=0$

$\beta=2 \quad \ell=3e$

$\beta>2 \quad \ell=+\infty$

(2) $$\lim_{x\to 0} \frac{\log(1+(2x^3+x^2))+\arctan(2x^3+x^2)+1-e^{4x^3}+\frac{1}{2}\operatorname{sen}(x^4)-2x^2}{(x^5+1)^{3/2}-\cos(x^3)}$$

N) $\log(1+(2x^3+x^2)) = \cancel{2x^3}+\cancel{x^2}-\frac{1}{2}(2x^3+x^2)^2+\cancel{\frac{1}{3}(2x^3+x^2)^3}+o(x^6)$

$\arctan(2x^3+x^2) = \cancel{2x^3}+\cancel{x^2}-\cancel{\frac{1}{3}(2x^3+x^2)^3}+o(x^6)$

$1-e^{4x^3} = -\cancel{4x^3}+o(x^5)$

$\frac{1}{2}\operatorname{sen}(x^4)-2x^2 = \frac{1}{2}x^4-\cancel{2x^2}+o(x^6)$

$= -\cancel{\frac{1}{2}x^4}-2x^5+\cancel{\frac{1}{2}x^4}+o(x^5) = \boxed{-2x^5+o(x^5)}$

D) $\cancel{1}+\frac{3}{2}x^5-\cancel{1}+\frac{x^6}{2}+o(x^6) = \boxed{\frac{3}{2}x^5+o(x^5)}$

$\boxed{\ell = -\frac{4}{3}}$

(3) $$\frac{\operatorname{sen}\left(\frac{x^4}{2}\right)+\arctan(3x^3+x^2)+1+\log(1+(3x^3+x^2))-e^{6x^3}-2x^2}{\cos(2x^4)-\sqrt{(1+x^5)}}$$

N: $\frac{x^4}{2}+(\cancel{3x^3}+\cancel{x^2})-\frac{1}{3}(3x^3+x^2)^3+\cancel{1}+(\cancel{3x^3}+\cancel{x^2})-\frac{1}{2}(3x^3+x^2)^2+\frac{1}{3}(3x^3+x^2)^3$

$\cancel{1}-\cancel{6x^3}-\cancel{2x^2}+o(x^5) = \boxed{-3x^5+o(x^5)}$

D) $\cancel{1}-\frac{2x^8}{2}-\cancel{1}-\frac{1}{2}x^5+o(x^5) = \boxed{-\frac{1}{2}x^5+o(x^5)}$

$\boxed{\ell = 6}$

(3) $\displaystyle\int_2^3 \frac{\sqrt{2x+1}}{x^2}\,dx$ Pongo $t=\sqrt{2x+1}$

$x = \frac{t^2-1}{2}$

$dx = t\,dt$

$$= \int_{\sqrt5}^{\sqrt7} \frac{4t\cdot t}{(t^2-1)^2}\,dt \underset{\text{per parti}}{=}$$

$$= \left[-\frac{2}{t^2-1}\cdot t\right]_{\sqrt5}^{\sqrt7} - \underbrace{\int_{\sqrt5}^{\sqrt7} -\frac{2}{t^2-1}\,dt}_{\mathrm{I}}$$

$$\frac{2}{t^2-1} = \frac{2}{(t-1)(t+1)}$$

Cerco A e B t.c.

$$\frac{2}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1}$$

$$= \frac{At+A+Bt-B}{(t-1)(t+1)}$$

$$\Leftrightarrow\ t(A+B)+A-B=2$$

$$\Leftrightarrow \begin{cases} A+B=0 \\ A-B=2 \end{cases} \quad \begin{cases} A=-B \\ -2B=2 \end{cases} \quad \begin{cases} A=1 \\ B=-1 \end{cases}$$

$$\Rightarrow \mathrm{I} = \int_{\sqrt5}^{\sqrt7} \frac{2}{t^2-1}\,dt = \int_{\sqrt5}^{\sqrt7} \left(\frac{1}{t-1} - \frac{1}{t+1}\right)dt$$

$$= \Big[\log(t-1) - \log(t+1)\Big]_{\sqrt5}^{\sqrt7}$$

$\boxed{\text{l'altro è analogo}}$

④ $f(x) = \sqrt{x+1}\ e^{|x|}$

DOMINIO : $x \geq -1 \Rightarrow [-1, +\infty)$

LIMITI : $\lim_{x \to +\infty} f(x) = +\infty$

SEGNO : $f(x) > 0 \quad \forall\ x > -1$

$f(-1) = 0$

DERIVATA PRIMA:

$$f'(x) = \frac{1}{2\sqrt{x+1}} e^{|x|} + e^{|x|} \cdot \operatorname{sgn} x \sqrt{x+1}$$

$$= \frac{1}{2\sqrt{x+1}} e^{|x|} \left[1 + 2\operatorname{sgn} x \cdot (x+1) \right]$$

$$= \begin{cases} \dfrac{1}{2\sqrt{x+1}} e^{|x|} \left[1 + 2x + 2 \right] & x > 0 \\ \dfrac{1}{2\sqrt{x+1}} e^{|x|} \left[1 - 2x - 2 \right] & x < 0 \end{cases}$$

se $\underline{x > 0}$ $\quad f'(x) \geq 0 \iff 2x + 3 \geq 0 \iff x \geq -3/2 \quad \Rightarrow f \nearrow \ \forall\ x > 0$

se $\underline{-1 \leq x < 0}$ $\quad f'(x) \leq 0 \iff -2x - 1 \geq 0 \iff x \leq -\frac{1}{2} \quad \Rightarrow f \nearrow$ in $[-1, -\frac{1}{2})$, $f \searrow$ in $(-\frac{1}{2}, 0]$

$x = -\frac{1}{2}$ pto di MAX RELATIVO

$x = -1$ pto di MIN. ASSOLUTO

• Pti di NON DERIVABILITA'

$x=0$: infatti $\lim_{x \to 0^+} f'(x) = \frac{3}{2}$

$\neq$

$\lim_{x \to 0^-} f'(x) = -\frac{1}{2}$

$x=-1$ infatti $\lim_{x \to -1^+} f'(x) = +\infty$